LEARNING AS ONE NATION

Expanded Project

Stat & Prob_18

SHS LEARNING ACTIVITY

Name: _____ **Score/Mark:** _____

Grade and Section: _____ **Date:** _____

Strand: ☐ STEM ☐ ABM ☐ HUMSS ☐ ICT (*TVL Track*)

Subject: ***Statistics & Probability***

Type of Activity: ☐ Concept Notes ☐ Skills: Exercise/Drill ☐ Illustration ☐ Performance Task ☐ Essay/Report ☐ Others: _____

Activity Title: Mean and Variance of Sampling Distribution of the Sample Mean

Learning Target: To compute the mean and variance of the sampling distribution of the sample mean.

References: *(Author, Title, Pages)*

1) Banigon, Jr. R. et al. "Statistics and Probability for Senior High School", pp.39-51;
2) L.A Sajulga, CVIF Learning Activities

If all possible samples of size n are randomly selected from a population of size N with mean μ and variance σ^2, then:

1. The **mean** of the sampling distribution of the sample mean is given by:

$$\boxed{\mu_{\bar{X}} = \mu}\ .$$

2. The **variance** of the sampling distribution of the sample mean is given by:

- $\boxed{\sigma^2{}_{\bar{X}} = \dfrac{\sigma^2}{n}}$ (for '*infinite*' population or when sampling is '*with replacement*')
- $\boxed{\sigma^2{}_{\bar{X}} = \left(\dfrac{\sigma^2}{n}\right)\left(\dfrac{N-n}{N-1}\right)}$ (for a '*finite*' population or when sampling is '*without replacement*')

Note that the $\left(\frac{N-n}{N-1}\right)$ is also called the *correction factor* since it will become closer to 1 if population N is large and sample size n is small.

We can verify these properties by referring to the example below.

Example: Suppose the probability distribution of $\bar{X}$ (from an infinite population with $\mu = 3$, variance, $\sigma^2 = 8/3$ and sample size $n = 2$) is given by:

$\bar{X}$	1	2	3	4	5
$P(\bar{X}) = f(\bar{X})$	$1/9$	$2/9$	$3/9$	$2/9$	$1/9$

The mean of the sampling distribution of the sample mean is:

$$\mu_{\bar{X}} = \sum \bar{X}\,(f(\bar{X})) = 1(1/9) + 2(2/9) + 3(3/9) + 4(2/9) + 5(1/9) = \frac{27}{9} = \boxed{\mathbf{3}} = \mu\,.$$

The variance of the sampling distribution of the sample mean is:

$$\sigma^2{}_{\bar{X}} = E(\bar{X}^2) - \mu_{\bar{X}}{}^2 = \left(1^2(1/9) + 2^2(2/9) + 3^2(3/9) + 4^2(2/9) + 5^2(1/9)\right) - 3^2 = \mathbf{\frac{4}{3}}$$

$$\text{or}\quad \sigma^2{}_{\bar{X}} = \frac{\sigma^2}{n} = \frac{8/3}{2} = \boxed{\mathbf{\frac{4}{3}}}\ .$$

Exercises:

1. Formulate the corresponding equation for the standard deviation, $\sigma_{\bar{X}}$, using the variance of the sampling distribution of the sample mean.
2. A school library has 2,000 books with a mean number of pages of 38 and a standard deviation of 3. If a random sample of 25 books is selected, find the mean, variance, and standard deviation of the sampling distribution of the sample mean.